EXTRA CHAPTER 3 REVIEW PROBLEMS

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True−False Quiz

1. $(f+g)'(x) = f'(x) + g'(x)$
2. $(fg)'(x) = f'(x)g'(x)$
3. $(f(g(x)))' = f'(g(x))g'(x)$
4. $(\sqrt{f(x)})' = \frac{f'(x)}{2\sqrt{f(x)}}$
5. $(f(\sqrt{x}))' = \frac{f'(x)}{2\sqrt{x}}$
6. If $y = e^2$, then $y' = 0$
7. $(10^x)' = x10^{x-1}$
8. $(\ln(10))' = \frac{1}{10}$
9. $\left(\tan^2 x\right)' = \sec^2 x$
10. $\left(\left|x^2+x\right|\right)' = |2x+1|$
11. The derivative of a polynomial is a polynomial
12. If $f(x) = (x^6 - x^4)^5$, then $f^{(31)}(x) = 0$
13. The derivative of a rational function is a rational function
14. An equation of the tangent line to the parabola $y = x^2$ at $(-2, 4)$ is $y - 4 = 2x(x+2)$
15. If $g(x) = x^5$, then $\lim_{x\to 2} \frac{g(x)-g(2)}{x-2} = 80$

Problems

3.R.68.

(a) By differentiating the double-angle formula

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

obtain the double-angle formula for sin

(b) By differentiating the addition formula

$$\sin(x+a) = \sin(x)\cos(a) + \cos(x)\sin(a)$$

obtain the addition formula for the cosine function

3.R.87. If the position of an object is $s(t) = Ae^{-ct}\cos(\omega t + \delta)$, find its velocity and its acceleration

3.R.94. Cobalt-64 has a half-life of 5.24 years.

(a) Find the mass that remains from a 100−mg sample after 20 years.
(b) How long would it take for the mass to decay to 1 mg

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3.R.99. A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a straight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

3.R.101. The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400-ft-tall building increasing when the angle of elevation of the sun is $\frac{\pi}{6}$.

3.R.105. A window has the shape of a square surmounted by a semicircle. The base of the window is measured as having width 60 cm witha possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error in computing the area of the window.

3.R.111. If $(f(2x))' = x^2$, find $f'(x)$

Answers

TRUE–FALSE.

(1) **TRUE**
(2) **FALSE**
(3) **TRUE**
(4) **TRUE**
(5) **FALSE** ($\frac{f'(\sqrt{x})}{2\sqrt{x}}$)
(6) **FALSE** (e^2 is a constant, so 0)
(7) **FALSE** ($y' = \ln(10)10^x$, exponential rule)
(8) **FALSE** ($\ln 10$ is a constant, so 0)
(9) **FALSE** ($2\tan(x)\sec^2(x)$)
(10) **FALSE** ($(2x+1)\frac{x^2+x}{|x^2+x|}$; Write $\left|x^2+x\right| = \sqrt{(x^2+x)^2}$ and use the chain rule)
(11) **TRUE**
(12) **TRUE** (f is a polynomial of degree 30, so its 31st derivative is 0)
(13) **TRUE** (By the quotient rule and (11))
(14) **FALSE** ($y - 4 = -4(x+2)$; it's not even the equation of a line!)
(15) **TRUE** ($= g'(2) = 5(2)^4 = 80$)

3.R.68.

(a) $\sin(2x) = 2\sin(x)\cos(x)$
(b) $\cos(x+a) = \cos(x)\cos(a) - \sin(x)\sin(a)$ (The important thing here is that you differentiate with respect to x, leaving a constant)

3.R.87.

$$v(t) = s'(t) = -Ace^{-ct}\cos(\omega t+\delta) - A\omega e^{-ct}\sin(\omega t+\delta) = -Ae^{-ct}\left(c\cos(\omega t+\delta) + \omega\sin(\omega t+\delta)\right)$$

$$a(t) = v'(t) = Ace^{-ct}\left(c\cos(\omega t+\delta) + \omega\sin(\omega t+\delta)\right) - Ae^{-ct}\left(-c\omega\sin(\omega t+\delta) + \omega^2\cos(\omega t+\delta)\right)$$

3.R.94. $y(t) = 100 \times 2^{-\frac{t}{5.24}}$

(a) $y(20) = 100 \times 2^{\frac{-20}{5.24}} \approx 7.1$ mg
(b) $t = \frac{5.24 \ln(100)}{\ln(2)} \approx 34.81$ years

3.R.99.

(1) $D^2 = x^2 + y^2$ (Typical Pythagorean theorem problem; Draw a right triangle in the shape of an L, and let x be the bottom side, y be the left-hand-side, and D be the hypothenus)
(2) $2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$
(3) $x = 3 \times 15 = 45$ (velocity $\times$ time), $y = 45 + 5 \times 3 = 60$ (initial height + velocity $\times$ time), $\frac{dx}{dt} = 15$, $\frac{dy}{dt} = 5$, and $D = \sqrt{x^2 + y^2} = \sqrt{45^2 + 60^2} = \sqrt{5625} = 75$ which gives:

$$\frac{dD}{dt} = \frac{45 \times 15 + 60 \times 5}{75} = 13$$

3.R.101.

(1) $\tan(\theta) = \frac{400}{x}$ (Typical trigonometry-problem. Draw another triangle in the shape of an L, let 400 be the left-hand-side, x be the bottom, and the angle on the right be θ)
(2) $\sec^2(\theta)\frac{d\theta}{dt} = -\frac{400}{x^2}\frac{dx}{dt}$
(3) $x = 400\sqrt{3}$ (redraw the same triangle, but this time with $\theta = \frac{\pi}{6}$), $\frac{d\theta}{dt} = -0.25$, and $\theta = \frac{\pi}{6}$, which gives:

$$\frac{dx}{dt} = 400$$

3.R.105. The area of the window is given by $y = x^2 + \frac{\pi}{2}\left(\frac{x}{2}\right)^2 = \left(1 + \frac{\pi}{8}\right)x^2$. Then:

$$dy = \left(1 + \frac{\pi}{8}\right)2x dx = \left(1 + \frac{\pi}{8}\right)(120)(0.1) = 12 + \frac{3\pi}{2} \approx 16.71$$

3.R.111. $f'(2x) = \frac{x^2}{2}$, so $f'(x) = \frac{\left(\frac{x}{2}\right)^2}{2} = \frac{x^2}{8}$